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On some interpolation methods

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Abstract

In part of this thesis, which comprises five papers, we study interpolation by the K- and J-methods of metric spaces. We further treat the problem of interpolation of Orlicz spaces. In this connection we make a comparison with the interpolation methods of Ovchinnikov, establishing also new results for interpolation of weighted L^p-spaces.

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Papers summarized in this dissertation.

A. J.Gustavsson, Interpolation of metric spaces, Technical report, Lund, (1971).

B. - , Metrization of quasi-metric spaces, Math. Scand. 35 (1974), 56-60.

C. J.Gustavsson and J.Peetre, Interpolation of Orlicz spaces, Studia Math. 60 (1977), 33-59.

D. J.Gustavsson, A function parameter in connection with interpolation of Banach spaces, Math. Scand. 42 (1978), 289-305.

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The theory of interpolation of abstract spaces has its roots in the classical convexity theorem of M.Riesz [24] dating from 1926, dealing with interpolation of linear operators between L^p-spaces. The original proof was a real variable proof. A new proof based on complex function theory was discovered in 1938 by his student Thorin [29]. Accordingly the theorem is often known as the Riesz-Thorin theorem. Another classical result is the Marcinkiewicz interpolation theorem [17] introducing the celebrated weak L^p-spaces. Published in 1939, shortly before the outbreak of the second world war, it was in oblivion until it was rediscovered in the middle 50's by Cotlar [3] and Zygmund [30].

The genesis of the abstract theory proper is from a period around 1960 when a number of interpolation methods of Banach spaces were invented by various authors, mainly with applications in partial differential equations in view. We mention e.g. the following "classics": Lions [15], Gagliardo [5], Lions-Peetre [16], Krein [14], Calderón [2]. Nowadays the most popular ones, beside the complex method of (Lions-)Calderón, are the K- and J-methods introduced by Peetre [19] in 1963. A detailed treatment of the theory and more references can be found in Bergh and Löfström [1].

The extension to more general spaces went more slowly. Quasi-normed spaces appear in the work of Kreĕ [13] but were fully exploited first in Holmstedt [10]. Locally convex spaces were treated by Girardeau [8], Deutsch [4] and Goullaouic [9]. It was however realized that the most fruiteful generalizations do not at all lie in the direction of topological vector spaces but on the metric side. Thus Peetre and Sparr [22] developed a theory of interpolation of normed Abelian groups, which has many interesting applications, e.g. to approximation theory. Interpolation of cones

was treated by Sagher [26]. In Peetre [20] there is then given a brief sketch of an extension of the K- and J-methods to metric spaces. This should potentially have many applications to non-linear analysis. Let us mention parenthetically that the famous Nash-Moser implicit function theorem is essentially a non-linear interpolation theorem. Some very interesting applications of non-linear interpolation are found in Tartar [27], [28]. As a further most recent extension of the K- and J-theory we mention Jawerth [11].

The first of the papers [A] is an attempt to put the ideas of Peetre [20] on interpolation of metric spaces, referred to above, on a more firm basis. However, several technical difficulties arose so we leave the subject in a still rather embryonic state. Also the subject of the short note [B], although not dealing directly with interpolation, was at least suggested in this connection.

Let us now return to Riesz's theorem. Rather than considering abstract generalizations we ask if it is possible to extend it directly to concrete classes of spaces. Many such results are known. Thus Rao [23], in connection with a question in ergodic theory, gave an extension of the Riesz-Thorin theorem when the end points spaces are Orlicz spaces. Due to the very nature of Thorin's method [29] one can in this way capture only a one-parameter family of intermediate spaces. There arises thus the question which is the most general Orlicz space which is an interpolation space with respect to two given Orlicz spaces.

In [C] (written together with Peetre) a partial solution of this problem is given. It is based on a new interpolation method in the abstract case, which we here somewhat informally term the $\pm$-method, having its origin in the work of Gagliardo [6] and

Peetre [21]. The ±-notation is motivated by a result of Rolewicz and Ryll-Nardzewski [25]. In unpublished work we also considered a dual method.

Already before [C] had appeared in print Ovchinnikov [18] published a very deep study, dealing with three interpolation methods, two of which are closely related to ours, while the third depends on Grothendieck's inequality. In passing he thereby also obtains a complete solution of the above interpolation problem for Orlicz spaces.

Our paper [D] was suggested by a faulty note by Kalugina [12]. In particular we establish a reiteration theorem which is applied to Orlicz spaces of the type $L^p(\log^+ L)^s$. It is also possible to give a duality theorem for Kalugina's method. A related result appears in the paper of Gapaillard and Merucci [7].

Finally [E] is devoted to a more close comparison of the two of Ovchinnikov's methods [18] and the ±-method of [C]. We also obtain some results for interpolation of weighted L^p-spaces.

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